PRACTICALLY STRUCTURAL ANALYSIS OF LARGE COOLING TOWERS

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ABSTRACT
For the structural analysis of large cooling towers exist different computation techniques developed to obtain the shell stresses with sufficient accuracy. Natural draft cooling towers are mostly designed as thin shells supported along the circumference by a system of columns. Hyperbolic, natural-draught cooling towers to be erected in seismically active zones are routinely analysed for earthquake excitation. Thus the earthquake safety of natural-draught cooling towers grows more important. The response of cooling towers assuming rigid-base translation excitation, can be approximated by beam elements with shear distortion and rotary inertia.

Keywords: cooling towers, practical analysis, high rise structures

INTRODUCTION
The cooling towers are rather peculiar structures being usually detached high-rise structures with the height determined by the recirculated water flow and the difference of the temperature that has to be extracted from the water. For currently used power plant capacities (<500 Mw) the cooling tower shells is about 100 m height with a corresponding basis diameter of 80 m and the wall thickness of about 16 cm. For large geometrical dimensions the wall thickness chosen for structural reasons required buckling safety as well as a sufficiently high natural frequency.

MATRIALS AND METHODS
Usually, the cooling towers are structures made up of rotational shells supported by inclinad columns forming a skeletal structure shaped as A, V or X letters ar, recently, by vertical columns placed at large distances (8 – 18 m) between them (Fig.1, a).

The cooling procedure (wet or dry cooling) to determine the geometrical dimensions. The size of the cooling tower is a direct function of the quantity of heat (Fig. 1, b).

Fig.1. The cooling tower:
- a. distances between vertical columns; b. size of cooling tower
Figures 2 and 3 to depict wind-buckling of cooling tower-model without and with upper edge beam.

The loading acting on a cooling tower is due to its own weight, wind, earthquake and temperature variation. The literature provides a large amount of both, practical techniques and refined methods for their analysis under axisymmetric loading. The same literature is still poor in methods for analysis of cooling towers under asymmetrical loading and efficient methods for their dynamic analysis. After several famous accidents, as that from FERRY BRIDGE (1965), when three cooling towers of 113 m height collapsed during a storm and that from IMPERIAL CHEMICAL INDUSTRIES Ltd (1973) when the cooling tower of ARDEER NYLON Plant fell down, the attention of the investigations has been drawn again to this type of structures and their dynamic behaviour. The dynamic analysis of cooling towers is a difficult task even if the modelling is as simple as using fing shaped finite elements. The paper presents an efficient free vibration analysis procedure using bar type finite elements with inner nodes.

1. **Structural discretization**
   
The shell of the cooling tower is discretized in isoparametric finite elements of bar type [1], with 3 or 4 inner nodes (Fig. 4).

The support of the shell is modelled as a flexible connection emphasizing flexibility along the three directions (Fig. 5). The analytical model of the discretized structure is based on Timoshenko type beams (Fig. 6), i.e. taking into account the shear deformations. Following the usual FEM
technique [2], [3], [4], [5], a dynamic condensation procedure eliminates the displacements of the inner nodes; 3 or 3 and 4 (Fig.4).

![Fig.5. Cooling tower structural model](image)

![Fig.6. Cooling tower Timoshenko beam model](image)

The interpolation functions for bar type finite elements [6], [7], [8], with three inner nodes are of the form:

\[ \Phi^1(r) = \frac{1}{2} r(1 - r) \]
\[ \Phi^2(r) = \frac{1}{2} r(1 + r) \]
\[ \Phi^3(r) = 1 - r^2 \]  

while in the case of finite elements with four inner nodes the interpolation functions reads:

\[ \Phi^1(r) = \frac{1}{2} r(1 - r) + \frac{1}{16} (-9r^3 + r^2 + 9r - 1) \]
\[ \Phi^2(r) = \frac{1}{2} r(1 + r) + \frac{1}{16} (9r^3 + r^2 - 9r - 1) \]
\[ \Phi^3(r) = 1 - r^2 + \frac{1}{16} (27r^3 + 7r^2 - 27r - 7) \]
\[ \Phi^4(r) = \frac{1}{16} (-27r^3 - 9r^2 + 27r + 9) \]  

The potential energy \( \pi \) due to both, bending and shear deformations is given by:
\[
\pi = \frac{1}{2} \int_0^L E_i I_i \left( \frac{d\beta}{dx} \right)^2 dx + \frac{1}{2} \int_0^L k_i G_i A_i \left( \frac{dy}{dx} - \beta \right)^2 dx
\]

where:
- \( I_i \) is the moment of inertia of the cross section,
- \( A_i \) is the cross section area,
- \( E_i \) is the longitudinal Young modulus,
- \( G_i \) is the transversal Young modulus,
- \( k_i \) is the shape coefficient,
- \( \beta \) is the rotational component due to bending only.

The equilibrium condition \( \delta \pi = 0 \) in the form of stationarity of function \( \pi \) with respecting \( x \) becomes

\[
\int_0^L E_i I_i \left( \frac{d\beta}{dx} \right)^2 \delta \left( \frac{d\beta}{dx} \right) dx + \int_0^L k_i G_i A_i \left( \frac{dy}{dx} - \beta \right) \delta \left( \frac{dy}{dx} - \beta \right) dx = 0
\]

Using the interpolation functions (1) for a three inner node finite element its displacements \( y \) and \( \beta \) may be expressed in terms of \( y_j \) and \( \theta_j \) as:

\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
\theta_1 \\
\theta_2 \\
\theta_3 \\
\end{bmatrix} =
\begin{bmatrix}
\Phi^1 & 0 & \Phi^2 & 0 & \Phi^3 & 0 \\
0 & \Phi^1 & 0 & \Phi^2 & 0 & \Phi^3
\end{bmatrix}
\begin{bmatrix}
y_j \\
\theta_j
\end{bmatrix}
\]

while the corresponding deformations device from the kinematics in the form:

\[
\begin{bmatrix}
\frac{dy}{dx} \\
\frac{d\beta}{dx}
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{3} \frac{\partial \Phi^i}{\partial x} y_j \\
\sum_{i=1}^{3} \frac{\partial \Phi^i}{\partial x} \theta_j
\end{bmatrix}
\]

where:

\[
\frac{\partial \Phi^i}{\partial x} = \begin{bmatrix}
\frac{\partial r}{\partial x} \\
\frac{\partial \Phi^i}{\partial r}
\end{bmatrix}
\]

and:

\[
\begin{bmatrix}
\frac{\partial r}{\partial x}
\end{bmatrix} = J^{-1}
\]

is the invers of the Jacobian matrix \( J \) of \( r \) in terms of \( x \).

Using (4) in a standard procedure the element stiffness matrix may be obtained. Having the fundamental matrices at the element level an analysis technique at the structural level can be performed. In the following, a nonlinear analysis procedure presented in [9] is applied. The
nonlinear equilibrium equations are solved via Newton-Raphson iterative technique which enjoy independence from the finite element type. The dynamic equilibrium equations are numerically integrated via either Newmark or Wilson methods. A computer program SUM01B has been developed to carry out the above mentioned computations. The program SUM01B using a unique vector and delivering the requested data through common blocks. The amount of memory required depends on the vector’s dimension [9].

2. Numerical example

The structure presented in figure 7 has been analyzed in [10].

![Fig.7. Column supported cooling tower [10]](image)

\[ R(z) = 35.5092 \sqrt{1 + \left( \frac{(106.9421 - x)}{88.8925} \right)^2} \]

\[ E = 2.758 \cdot 10^7 \, kN \, / \, m^2 \]

\[ h = 0.3048 \, m \]

\[ \vec{d} = 0.167 \]

\[ \rho = 2.4tf \, / \, m^3 \]

44 INCLINED (19°) COLUMNS

WITH 1.3208 \cdot 0.6906 m² CROSS SECTION

Bar type finite elements with three nodes have been used and the curvature has been dealt with by taking there are coefficients \( k_y = 1.5 \cdot 10^6 \, tf \, / \, m \) and \( k_{\theta} = 1.99 \cdot 1010tf \, / \, m \). The free vibration analysis lead to the conclusion that the first eigen modes are of transversal vibration type. Indeed, the frequency of the transversal vibrations is much smaller than those of either longitudinal or torsional vibrations [12], [13], [14], [15]. For the above mentioned structure, modeled using eight finite elements (Fig.8) the obtained frequency of the transversal vibrations is 2.472 Hz versus 2.296 Hz given in [10]; a difference of about 7.7%.
CONCLUSIONS

* The theoretical backgrounds and numerical example used the Timoshenko type beams model are presented.
* Modelling the cooling towers by using bar type finite elements having three or four inner nodes offers an efficient vibration analysis which emphasizes simplicity and an acceptable accuracy for the practical engineering needs, [16], [17].
  * The presented free vibration analysis technique may, therefore, be used in a seismic analysis using the enforced aseismic design needs, [18], [19], [20].
  * The tall shells for natural draft cooling towers show the necessity for continuous scientific research, [21], [22], [23].

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