ASPECTS OF VOLUME CALCULATION

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ABSTRACT
This article deals with the problem of calculating volumes using the digital terrain model. Large majority of current programs for calculating volumes uses digital terrain model for volume calculation. Data acquisition using the terrestrial laser scanner is one of the newest techniques of data acquisition to achieve digital terrain model. In this article we use a digital terrain model obtained after measurements using a terrestrial laser scanner and we make a comparative study between the classical treatment using the Simpson’s formula and the automatic calculating of volume with a computer program.

Keywords: laser scanner, DTM, three-dimensional modelling, volume calculation

INTRODUCTION
The current programs for calculating volumes use as entry data the digital terrain model. There are several methods to acquire elevation data for digital terrain model: a) ground survey methods using total stations; b) photogrammetric methods based on the use of stereoplotting instruments; c) graphics digitizing methods using tablet digitizers; d) data acquisition using laser scanning technique.

At first using of an automatic calculation volumes program, it is important to solve at least one case study, using the classic version, to be compared with the program that automatically generates volumes.

MATERIALS AND METHOD
Laser Scanning is a new geodetic technique, whereas the geometry of a structure is measured (more or less) completely automated and reflector less with high precision and velocity. The origin result is a so-called point cloud [1]. The laser scanner records the points three-dimensionally by measuring the horizontal and vertical angle, as well as the spatial distance for each point. The coordinates of the points are obtained in a specific Cartesian system of the scanner using simple trigonometric functions. The horizontal and vertical angles are modified automatically in pre-established intervals.

An important application is the creation of digital terrain model DEM (Digital Elevation Model). The software used is Cyclone (Leica) and permits obtaining the TIN model (Triangulated Irregular Network), which is a spatial network of triangles, created as a Delaunay algorithm [2]. This method is being used to an ever-increasing extent in terrain modelling. The reasons for this development are that every measured data point is being used and honoured directly, since they form the vertices of the triangles used to model the terrain, from which the height of additional points may be determined by interpolation and the construction of contours undertaken. Furthermore, the use of triangles offers a relatively easy way of incorporating break-lines, fault lines, etc. Any triangular-based approach should attempt to produce a unique set of triangles that are as equilateral as possible and with minimum side lengths. Using digital terrain model it can perform calculations on cut and fill volumes for a given site.
At first using of an automatic calculation volumes program it is important to solve at least one case study, using the classic version, to be compared with the program that automatically generates volumes.

1. Volume calculation using Simpson’s formula

In numerical analysis, Simpson’s formula is a method of numerical integration, numerical approximation of defined integrals [3]. Simpson’s formula calculates an approximation of the integral function \( f(x) \) (in blue) by a second degree polynomial, \( P(x) \) (in red), which takes the same values of the function \( f(x) \) at the ends \( a \) and \( b \) and at the midpoint \( m \) (\( m = (a + b)/2 \)).

\[
\int_{a}^{b} f(x) \, dx \approx \frac{b-a}{6} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]
\]

(1)

\[
P(x) = f(a)\frac{(x-m)(x-b)}{(a-m)(a-b)} + f(m)\frac{(x-a)(x-b)}{(m-a)(m-b)} + f(b)\frac{(x-a)(x-m)}{(b-a)(b-m)}
\]

(2)

If the approximation interval \([a, b]\) is in some sense "small", Simpson’s formula will provide a suitable approximation for the exact integral. By small means that the function to be integrated is relatively smooth on \([a, b]\). For such a function, a second degree polynom, as used in Simpson’s formula gives good results. However, it often happens that the function we are trying to integrate is not smooth throughout the range. This means that the function is powerful swing or there are missing function values at certain points. In these cases, Simpson’s rule may give poor results. One way to solve this problem is to divide the interval \([a, b]\) into a number of subintervals. Simpson’s formula will be applied to each subinterval and summing the results will get an approximation of the integral over the entire interval. This is called Simpson’s composite formula. Assuming that \([a, b]\) is divided into \( n \) subintervals, where \( n \) is an even number, then Simpson’s rule is as follows:

\[
\int_{a}^{b} f(x) \, dx \approx \frac{h}{3} \left[ f(x_0) + 2\sum_{j=1}^{n-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(x_n) \right]
\]

(3)

where: \( x_j = a + jh \) for \( j = 0,1,...,n - 1,n \) with \( h = (b - a) / n \); in particular, \( x_0 = a \) and \( x_n = b \).
Therefore the steps are: selecting an even number of intervals (odd number of sections) and determine the area of each section on the profile of the section. The volume can be calculated by the Simpson’s formula:

\[
\text{Volume} = \frac{h}{3} \left( \text{area of first section} + \text{area of last section} + \text{twice the amount of odd sectional areas} + \text{four times the amount of even sectional areas} \right)
\]

(4)

2. Volume calculation using the cyclone software

Based on TIN model and a reference plane it can be calculated the volumes of different enclosures. The first step is to generate a reference plane (Fig.2), in the "Tools"/"Reference plane". It can be defined by the axes of the coordinate system and an origin point and the software calculates the volume of the enclosure from the reference plan to the surface generated by TIN model. For reference plane may be imposed grid size, grid extension to cover the desired volume to be calculated, color and thickness grid. Volume is calculated in units set to section "Edit"/"Preferences"/"Units".

![Fig.2. Reference Plane Parameters](image)

3. Filling and excavation volume calculation

Based on TIN model and a chosen reference plane can be calculated volumes of filling and excavation in order to choose the most convenient situation in terms of volume of excavation, filling volume, the surface on which digging and area that will be filled.

![Fig.3. Choosing grid parameters](image)
The position of the reference plane is set in the menu "Tools" / "Reference plane" from the "Tools" / "Measure" / "Measure surface deviation". The program will generate (Fig. 3), based on square grid with edge length set, a table with grid square corners position, the heights of points, the reference height and the difference between the reference height and the field height.

Differences are positive and negative depending on the position of the reference plane. In each position of the reference plane, using the command "Tools" / "Measure" / "TIN volume" it can get cut and fill volumes and surfaces to be performed excavation and filling. (Fig. 4). Through successive attempts it can be obtained the best solution in technical and economical terms for the excavation and filling areas.

To check the software we determined an odd number of sections in the interested zone. Using digital terrain model, we obtained the surfaces of profiles using digital terrain model already created. We apply the Simpson's formula and we checked the volume calculation. The results showed that automatic volume calculation program is working correctly.

CONCLUSIONS

In calculating volumes obtained from computer programs automatically based on three-dimensional coordinate points are two important aspects: one is related to the mode of data collection in the field to provide sufficient three-dimensional points representing terrain configuration, depending on the desired accuracy to obtain volume and the second aspect is given by the decision to establish that the method for volume calculation presents correct results. Engineer's task is to establish and verify the values provided by automatic calculation of volumes programs. The first step is to verify, at least at first use of the software, the results obtained with a classical computing method.

REFERENCES